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AN ANALYSIS OF TRANSIENT FLOW OF HEAT AND MOISTURE DURING DRYING OF GRANULAR BEDS

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Abstract—The transient process of drying granular materials in fixed packed beds is analysed, and equations expressing moisture contents and temperatures of material and drying medium as functions of time and location in the bed are derived. While moisture content of the granules may be expressed in terms of ordinary exponential functions of these variables, humidity and temperature of the drying medium additionally require numerical solutions. The theory is developed for materials drying in four stages—one stage at constant rate, one at increasing rate, and two at falling rates—and should be applicable to numerous products in the chemical and food industries, with wider applicability if suitably modified. Application of the theory is demonstrated for through-drying of a granular bed of wheat, using published data to evaluate transfer parameters appropriate to special drying conditions and employing a CDC3200 digital computer to calculate the moisture content of grains as a function of time and location. The Fortran programme written for this computation may be used to predict the progress of drying other materials within the category of the present analysis.

NOMENCLATURE

C, specific heat (humid heat for the gas) [kcal/kg degC];
$(CR)(0)$, constant rate at $y = 0$;
$(CR)(y)$, constant rate at $y > 0$;
(FR), falling rate;
G, mass velocity of gas [kg dry gas/h m²];
H, specific humidity of gas [kg H_2O/kg dry gas];
$(IR)(y)$, peak increasing rate at $y > 0$;
k, slope of drying curve [h^{-1}];
K, volumetric coefficient of mass transfer, [kg H_2O/h m³ (kg H_2O/kg dry air)];
t, time [h];
T, temperature [°C];
U, volumetric coefficient of heat transfer [kcal/h m³ degC];
V, velocity of drying medium [m/h];
X, moisture content [kg H_2O/kg dry material];
y, height [m].

Constants defined in text (equation numbers in parenthesis):
a (B.2); A (16); b (B.3); B (11); D (A.4); E (C.4); L (C.7); M (32); q (B.2); r (B.3); u_a (29); u_b (C.3); v_a (30); v_b (C.10).

Greek symbols
α, defined by equation (5);
β, defined by equation (8);
λ, latent heat of evaporation of water [kcal/kg];
ρ, density (bulk density on dry weight basis for solids) [kg/m³].

Subscripts
c, constant rate;
e, equilibrium;
g, gas;
i, point of gas entry;
I, increasing rate;

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l,	liquid;
s,	solids;
o,	initial conditions;
1,	first critical moisture content;
2,	second critical moisture content.

1. INTRODUCTION

DURING the drying of granular material in stationary beds, temperatures and moisture contents of both the material and the drying medium change continuously with time and also with location in the bed. Moreover, the rates at which these changes occur depend mainly on the composition, structure and packing of the granules, and on the composition, flow and initial state of the drying medium. Heat transfer between gas streams and *dry* granular beds—a subject relevant to part of the present study—has already been analysed in terms of a mechanism for which mathematical solutions have been presented graphically [1, 2]. Wider applicability of this earlier work has since been attempted by graphical solution of the basic equations in differential form [3] and in finite difference form [4], and also by nomographical presentation of approximate error functions [5].

It is well known that heat and mass transfer across a boundary layer can be treated alike, the transfer rates being functions of driving forces expressed as temperature differences or concentration differences, respectively. Thus previous studies of pure heat transfer may provide a basis for the present investigations relating to heat and mass transfer. However, the results of the earlier work are clearly not completely applicable to drying in granular beds, since:

1. The latent heat required for vaporization of water is usually considerably larger than the sensible heat of the wet material during a large part of the drying period.
2. During the constant-rate stage of drying, when nearly all heat supplied is utilized for the vaporization of water, the solids temperature remains approximately at the wet bulb temperature of the drying medium [6–9]; consequently, at this stage temperature differences cannot provide useful data for prediction purposes.
3. With the above theoretical approach, transition from the constant-rate stage to changing-rate stages of drying results in mathematical discontinuities.

Although a simplified analysis of moisture transfer during adiabatic batch drying has been made by van Meel [10], a detailed study of the simultaneous transfer of heat and moisture has apparently not yet been published. The present study is directed towards this problem, and while it is hoped that it will contribute towards a better understanding of basic principles of drying, the resulting physical relationships should enable process engineers to predict the progress of drying and thereby anticipate thermal damage to particular products. Analyses of changing conditions in the drying medium are of especial interest in the drying of foods, where, in order to limit microbial growth, strict control of environmental temperatures and humidities are required [11–13].

In the following general analysis of drying a stationary granular bed with a gas, there will for each phase be derived expressions for time-dependent changes of moisture contents and temperatures at various locations in the bed. Mathematical formulations will be based on one-dimensional changes, i.e. conditions affecting the states of the solids and gases will be assumed not to vary in planes perpendicular to the direction of gas flow. This assumption proved reasonably accurate for similar processes, such as heat and mass transfer of heterogeneous chemical reactions in granular beds [14]. It will be assumed also that between the gases and solids the rates of heat and mass transfer remain proportional to differences in temperature and humidity, respectively. Strictly, this assumption limits application of the theory to cases where transfer coefficients are constant, and it is well known that these coefficients vary to some extent with process conditions. Nevertheless, it has been established in practice—at least for heat transfer [3]—that instead of the exact variable function an average constant value can be used as a first-order approximation. The same is expected to hold true for mass transfer.

For industrially important materials, e.g. calcium carbonate, coke and asbestos [15], clay and kaolin–sand mixtures [16], and pumice [17], as well as for foods and grain [18], small samples dry over approximately three consecutive stages, as illustrated by the idealized plot of rate of drying as a function of moisture content (Fig. 1). These characteristics, generally based on the drying of *shallow* samples under constant conditions of the drying media, may be determined either by carefully designed laboratory methods [15] or by automatic recording instruments [19].

When drying *deep beds* of the same materials, a fourth stage of drying will be apparent; this will be discussed in Section 4. The following theory, limited to these four stages, requires a special analysis of the transfer of heat and moisture for each individual stage. Should it be required to handle materials whose drying characteristics are represented by more than four stages, the technique used here is still applicable, because, as will become apparent later, additional stages may readily be incorporated in the analysis. However, the resulting equations would increase in complexity and the accuracy would decrease.

2. RATE EQUATIONS

The system considered below is a vertical, packed column, whose load of wet solids is being dried by an ascending flow of a gaseous drying medium. For a cross-sectional area of unity for the column when empty, a material balance for the transfer of moisture from the wet solids to the drying medium yields the following differential equation:

$$\frac{dX}{dt} = -\frac{K}{\rho_s}(H_s - H_g) = -\frac{G}{\rho_s}\frac{dH_g}{dy}. \qquad (1)$$

In the same drying zone an energy balance for the transfer of heat from the drying medium to the wet solids gives the approximate differential equation (heat losses are neglected):

$$\frac{dX}{dt} - \frac{C_l X}{\lambda}\frac{dT_s}{dt} - \frac{C_s}{\lambda}\frac{dT_s}{dt} = -\frac{U}{\lambda\rho_s} \times (T_g - T_s) = \frac{C_g G}{\lambda\rho_s}\frac{dT_g}{dy}. \qquad (2)$$

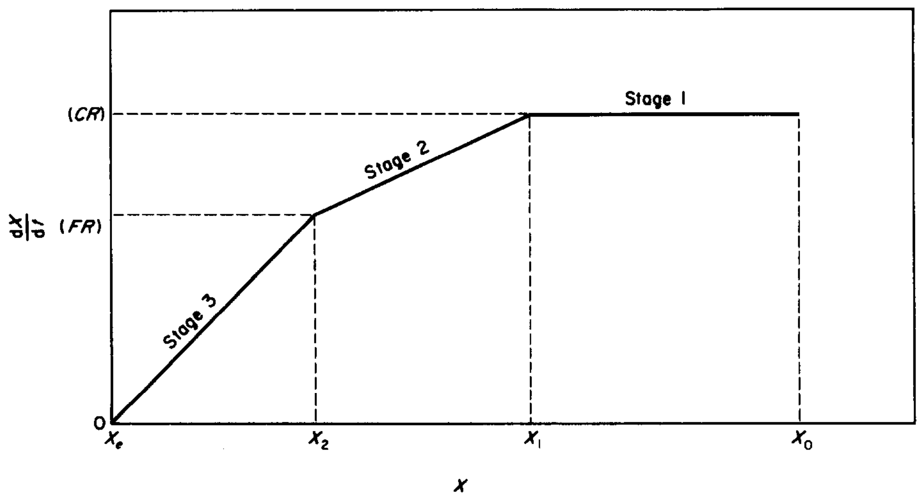

FIG. 1. Stagewise progress of drying shallow samples.

Coefficients of heat and mass transfer, here expressed on a volumetric basis, are directly related to the corresponding coefficients on an area basis through the specific surface area of the packed material. Equations (1) and (2), in conjunction with generalized drying characteristics, will now be used to derive expressions for changes in moisture contents, humidities, and temperatures during each of the four stages of drying. (Note that throughout this paper negative numerical values are implied in all symbols that express rates of drying.)

3. DRYING AT CONSTANT RATE

The following analysis is carried out on the assumption that a gaseous drying medium of constant temperature and humidity is employed. Under these conditions, commonly aimed at for controlled drying in industry, T_s remains approximately at the wet bulb temperature of the drying medium, and H_s at the surface of the solid remains nearly equal to the saturation humidity at T_s.

3.1. *Temperature variation*

On the basis of the two right-hand terms of equation (2) the temperature change in the drying medium may be expressed by:

$$\frac{dT_g}{dy} = -\frac{U}{C_g G}(T_g - T_c), \tag{3}$$

in which T_c is the nearly constant temperature of the solids. The solution of equation (3) is:

$$T_g = T_c + (T_i - T_c)\,e^{-\alpha y}, \tag{4}$$

where T_i is the constant temperature at the gas entrance to the bed, and

$$\alpha = \frac{U}{C_g G}. \tag{5}$$

3.2. *Humidity variation*

From the two right-hand terms of equation (1), the differential equation expressing humidity changes in the drying medium is

$$\frac{dH_g}{dy} = \frac{K}{G}(H_c - H_g), \tag{6}$$

in which H_c is the nearly constant saturation humidity near the surface of the solids. The solution of equation (6) is

$$H_g = H_c + (H_i - H_c)\,e^{-\beta y}, \tag{7}$$

where H_i is the humidity of the gas at the entrance to the bed and

$$\beta = \frac{K}{G}. \tag{8}$$

3.3. *Moisture content variation*

An equation for the change of moisture content in the solids may be obtained from the two left-hand terms of equation (1), modified for the constant-rate stage of drying:

$$\frac{dX}{dt} = -\frac{K}{\rho_s}(H_c - H_g). \tag{9}$$

Combining equations (7) and (9) leads to the partial differential equation:

$$\left(\frac{\partial X}{\partial t}\right)_y = -B\,e^{-\beta y}, \tag{10}$$

where

$$B = \frac{K}{\rho_s}.(H_c - H_i). \tag{11}$$

The solution of equation (10) is

$$X = X_o - Bt\,e^{-\beta y} \tag{12}$$

with initial conditions $X = X_o$ at $t = 0$ for all values of y.

From equation (2) the change in moisture content may also be expressed in terms of heat-transfer data, as follows:

$$\frac{dX}{dt} = -\frac{U}{\lambda \rho_s}(T_g - T_c). \tag{13}$$

From (4) and (13),

$$\left(\frac{\partial X}{\partial t}\right)_y = -\frac{U}{\lambda \rho_s}(T_i - T_c)\,e^{-\alpha y}, \tag{14}$$

the solution for the same initial conditions as equation (12) being:

$$X = X_o - At\,e^{-\alpha y}, \tag{15}$$

where

$$A = \frac{U}{\lambda \rho_s}(T_i - T_c). \tag{16}$$

For the following reasons it is desirable to describe the progress of drying in terms of either of the two sets of physical parameters of equations (12) and (15), respectively:

1. Either heat-transfer coefficients or mass-transfer coefficients (but less frequently both) could be available from published work on the product concerned.
2. Determinations of transfer data for unfamiliar products can for some practical purposes be limited to heat-transfer data, which are the more accurately measured [20].
3. Only this arrangement enables one to study coupling between heat and mass transfer during drying and to define the course of the process in terms of unmixed transfer parameters, as in the next sections.

4. DRYING AT CHANGING RATES

An analysis will be made of the most general case of drying, viz. when the initial moisture content X_o of the wet material is greater than the first critical moisture content X_1. The drying characteristics for this situation (see Fig. 2) may be represented by one constant-rate stage $(CR)(y)$, one increasing-rate stage, and two falling-rate stages of the material being dried, and the drying medium goes through the following changes: during the constant-rate stage, the temperature and humidity of the drying medium at any given point in the bed depend only on the height of the point from the bottom of the bed; but after the bottom layer of packed material has reached the critical moisture level X_1, the psychrometric condition of the drying medium becomes a function of time also. Subsequently the rate of drying of the bottom layer gradually diminishes and the ascending drying medium gradually becomes less cooled and less humidified; only where the drying medium enters at the bottom of the bed are its temperature and humidity constant at all times (as specified for the present analysis). This gradual

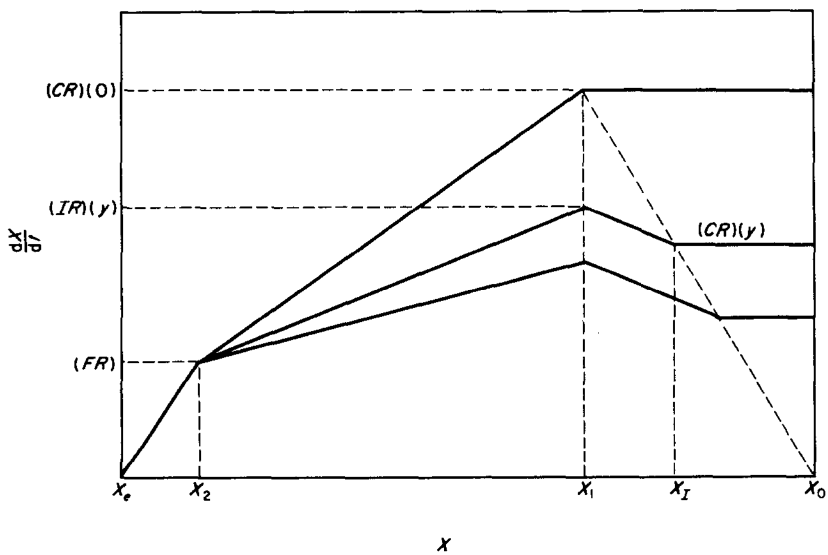

FIG. 2. Stagewise progress of drying deep packed beds.

improvement in the drying power of the drying medium is the reason for the increasing-rate stage of drying of material within the body of the bed.

4.1. *Increasing-rate stage*

(a) *Moisture variation*. The moisture content of material at any level of the bed is reduced from X_I to X_1 (Fig. 2) at the rate:

$$\frac{\mathrm{d}X}{\mathrm{d}t} = (CR)(y) + k(X - X_I), \tag{17}$$

which was derived from an analytical expression for slope k of the increasing-rate curve. In practice, slope k does not appear to change greatly for various levels of the bed [21] and will be assumed constant for the present analyses. In general, accordingly, an average slope should be estimated from measurements at a middle level of the bed. The solution of equation (17) is (see Appendix A):

$$X = X_o + \frac{B}{k}\mathrm{e}^{-\beta y}\,[D - \exp\{k(t - t_I)]\}. \tag{18}$$

(b) *Humidity variation*. An expression for changes of humidity in the drying medium during the increasing-rate stage may be derived from the two right-hand terms of equation (1):

$$\frac{\mathrm{d}H_g}{\mathrm{d}y} + \beta H_g = \beta H_s. \tag{19}$$

Also, during the increasing-rate stage of drying the humidity H_s of air at the surface of the solids is approximately equal to the humidity of saturated air at the surface temperature T_s, which increases as local conditions (temperature and humidity) of the drying medium gradually improve. H_s is now therefore a function of time and height and must be determined for substitution into equation (19). The subsequent solution of equation (19) is given in Appendix B, producing

$$H_g = r + bX_o + \left[H_i - r - bX_o + \frac{\beta bB}{k}y \times \{D - \exp[k(t - t_I)]\}\right]\mathrm{e}^{-\beta y} \tag{20}$$

(c) *Temperature variation*. An expression for the temperature of the drying medium, also changing with time and position in the granular bed, may be derived from the two right-hand terms of equation (2):

$$\frac{\mathrm{d}T_g}{\mathrm{d}y} + \alpha T_g = \alpha T_s \tag{21}$$

together with the linear relations between solids temperature and moisture content (discussed in Appendix B):

$$T_s = q + aX. \tag{22}$$

It is now desirable to express X in terms of heat-transfer data, viz:

$$X = X_o + \frac{A}{k}\mathrm{e}^{-\alpha y}\left\{D - \exp[k(t - t_I)]\right\}, \tag{18a}$$

the derivation being analogous to that of equation (18). Combining equations (18a), (21) and (22) leads to:

$$\left(\frac{\partial T_g}{\partial y}\right)_t + \alpha T_g = \alpha q + \alpha a\left[X_o + \frac{A}{k}\mathrm{e}^{-\alpha y} \times \{D - \exp[k(t - t_I)]\}\right]. \tag{23}$$

The solution of equation (23) [mathematically equivalent to equation (B.4), which was the basis for equation (20)] may be obtained by replacing the mass-transfer parameters β, b, B, H_g, H_i, r in equation (20) by the corresponding heat-transfer parameters α, a, A, T_g, T_i, q, giving:

$$T_g = q + aX_o + \left[T_i - q - aX_o + \frac{\alpha aA}{k}y \times \{D - \exp[k(t - t_I)]\}\right]\mathrm{e}^{-\alpha y}. \tag{24}$$

4.2. *First falling-rate stage*

(a) *Moisture variation*. During the first falling-rate stage of drying, the moisture content of the granular bed is reduced from X_1 to X_2, as shown in the drying characteristics of Fig. 2.

During drying of materials with a high initial moisture content, (FR) may be considered to be nearly independent of y, i.e. the drying rate is approximately (FR) in any level of the bed once the moisture content in that level has been reduced to X_2 [21]. However, as shown numerically in Section 5 and analytically in Appendix D, the moisture content will not at any given moment be equal to X_2 in all levels, but could be considerably lower in the bottom regions when it is equal to X_2 in the upper regions. These regions would then receive drying medium that had been only slightly cooled and humidified during its ascent through the bed; this, in fact, is the reason why (FR) is nearly constant.

The rate of drying in this stage may be described by the differential equation:

$$\frac{dX}{dt} = (FR) + \left[\frac{(IR)(y) - (FR)}{X_1 - X_2}\right](X - X_2), \quad (25)$$

which was derived from the slope of the drying curve extending from X_1 to X_2. The solution of equation (25), worked out in detail in Appendix C, is:

$$X = X_2 + \frac{(FR)}{u_b} + v_b^{\frac{u_b}{k}}\left[X_1 - X_2 - \frac{(FR)}{u_b}\right] \times \exp\left[-u_b(t - t_I)\right], \quad (26)$$

the transformations expressed by u_b and v_b being defined in the same appendix.

(b) *Humidity variation.* Derivation of an expression for changes of humidity in the drying medium may now be based on equations (19, 26, B.3), using values for b, r, and X in equation (26) that are applicable in the first falling-rate stage. This leads to the partial differential equation:

$$\left(\frac{\partial H_g}{\partial y}\right)_t + \beta H_g = \beta(r + bX_2) + \frac{\beta b(FR)}{u_b} + \beta b v_b^{\frac{u_b}{k}}\left[X_1 - X_2 - \frac{(FR)}{v_b}\right] \times \exp\left[-u_b(t - t_I)\right], \quad (27)$$

the initial conditions (i.e. when $y = 0$) being:

$$H_g = H_i \quad \text{when} \quad u_b = \frac{BD + E}{X_1 - X_2} \quad \text{and} \quad v_b = \frac{L}{B} + D.$$

This equation can be solved numerically, e.g. by the Runge–Kutta method [24], on a digital computer [25], and can be programmed as a sub-routine in a main programme of the type used in the present study.

(c) *Temperature variation.* A relationship expressing changes in temperature of the drying medium with time and position may be derived on the basis of equations (21) and (22). The linear relation between temperature and moisture content of solids, equation (22), assumed for the small changes of T_s expected during the increasing-rate stage of drying discussed above, has actually been confirmed theoretically and experimentally for the larger changes of T_s expected during drying at falling rates [26]. The expression for temperature changes:

$$\left(\frac{\partial T_g}{\partial y}\right)_t + \alpha T_g = \alpha(q + aX_2) + \frac{\alpha a(FR)}{u_a} + \alpha a v_a^{\frac{u_a}{k}}\left[X_1 - X_2 - \frac{(FR)}{u_a}\right] \times \exp\left[-u_a(t - t_I)\right], \quad (28)$$

in which:

$$u_a = \frac{AD\,e^{-\alpha y} + E}{X_1 - X_2} \quad (29)$$

and

$$v_a = \frac{L}{A}e^{\alpha y} + D, \quad (30)$$

may be solved by the same method as that indicated for equation (27).

4.3. *Second falling-rate stage*

(a) *Moisture variation.* During this stage of drying—the hygroscopic stage—the moisture content of the granules is reduced from X_2 to X_e at a rate which, for equal values of X, tends to be the same for all locations in the bed [27].

Also:

$$\frac{dx}{dt} = M(X - X_e) \tag{31}$$

in which:

$$M = \frac{(FR)}{X_2 - X_e}. \tag{32}$$

Equation (31) was solved in Appendix D and gave:

$$X = X_e + (X_2 - X_e)\, v_b^{-\frac{M}{k}} \times \left[1 - \frac{u_b(X_1 - X_2)}{(FR)}\right]^{-\frac{M}{u_b}} \exp\left[M(t - t_I)\right]. \tag{33}$$

(b) *Humidity variation.* An expression for humidity changes of the drying medium during the hygroscopic stage may be derived by combining equations (19, 33, B.3), giving:

$$\left(\frac{\partial H_g}{\partial y}\right)_t + \beta H_g = \beta r + \beta b \left\{X_e + (X_2 - X_e)\, v_b^{-\frac{M}{k}} \times \left[1 - \frac{u_b(X_1 - X_2)}{(FR)}\right]^{-\frac{M}{u_b}} \times \exp\left[M(t - t_I)\right]\right\}. \tag{34}$$

Strictly, the linear relation expressed by equation (B.3) holds good only when H_s is the saturation humidity at T_s. Although the drying medium near granular surfaces is not saturated during the hygroscopic stage of drying, equation (34) will be assumed valid. Its practical application would require that b and r be determined for the particular material to be dried, and in the case of large variations in the temperature of the material it might be necessary to determine various values for b and r over various intervals of moisture content of the material. Equation (34) may be handled in the way indicated for equation (27).

(c) *Temperature variation.* In accordance with the above discussions, temperature changes of the drying medium may be derived by analogy with equation (34):

$$\left(\frac{\partial T_g}{\partial y}\right)_t + \alpha T_g = \alpha q + \alpha a \left\{X_e + (X_2 - X_e)\, v_a^{-\frac{M}{k}} \times \left[1 - \frac{u_a(X_1 - X_2)}{(FR)}\right]^{-\frac{M}{u_a}} \times \exp\left[M(t - t_I)\right]\right\}, \tag{35}$$

which is solved as indicated for equation (27).

5. NUMERICAL EXAMPLE

Application of the above theory will now be demonstrated by using equations (12, 18, 26, 33) to calculate changes in moisture content during *large scale* drying of grain (wheat) with warm air. Numerical values for the following constants in these equations were mainly determined on the basis of *small scale* laboratory experiments carried out elsewhere [21] at an air temperature of 20·4°C, an air humidity of 55 per cent r.h. and an air velocity of 1·38 m/s.

$(FR) = -0{\cdot}10$ kg/kgh, from [21];
$k = 8{\cdot}8\ h^{-1}$, from [21];
$X_o = 0{\cdot}75$ kg/kg, from [21];
$X_1 = 0{\cdot}65$ kg/kg, from [21];
$X_2 = 0{\cdot}55$ kg/kg, from [21];
$X_e = 0{\cdot}10$ kg/kg, from [21];
$B = \frac{K}{\rho_s}(H_c - H_i) = 1{\cdot}6$ kg/kg h;
$D = 1 - kt_I = 0{\cdot}449$ (dimensionless);
$E = L + (FR) = 0{\cdot}78$ kg/kg h;
$G = V\rho_g = 5930$ kg/h m²;
$K = -\frac{(dX)^*}{(dt)_c}\frac{\rho_s}{(H_c - H_i)} = 47\,500$ kg/hm³ (kg/kg);
$L = k(X_o - X_1) = 0{\cdot}88$ kg/kg h;

* $(dX)/(dt)_c$ = rate of drying in the constant rate stage.

Table 1. *Values of X as a function t and y*

t, h	y, m: 0	0·10	0·20	0·30	0·40	0·50	0·60	0·80	1·00	1·50	2·00
0	0·7500	0·7500	0·7500	0·7500	0·7500	0·7500	0·7500	0·7500	0·7500	0·7500	0·7500
0·01	0·7340	0·7428	0·7468	0·7485	0·7493	0·7497	0·7499	0·7500	0·7500	0·7500	0·7500
0·02	0·7180	0·7356	0·7435	0·7471	0·7487	0·7494	0·7497	0·7499	0·7500	0·7500	0·7500
0·03	0·7020	0·7284	0·7403	0·7456	0·7480	0·7491	0·7496	0·7499	0·7500	0·7500	0·7500
0·04	0·6860	0·7212	0·7371	0·7442	0·7474	0·7488	0·7495	0·7499	0·7500	0·7500	0·7500
0·05	0·6700	0·7141	0·7338	0·7427	0·7467	0·7485	0·7493	0·7499	0·7500	0·7500	0·7500
0·06	0·6540	0·7069	0·7306	0·7413	0·7461	0·7482	0·7492	0·7498	0·7500	0·7500	0·7500
0·07	0·6385	0·6994	0·7272	0·7398	0·7454	0·7479	0·7491	0·7498	0·7500	0·7500	0·7500
0·08	0·6252	0·6913	0·7236	0·7382	0·7447	0·7476	0·7489	0·7498	0·7500	0·7500	0·7500
0·09	0·6138	0·6826	0·7197	0·7364	0·7439	0·7473	0·7488	0·7498	0·7499	0·7500	0·7500
0·10	0·6040	0·6730	0·7154	0·7344	0·7430	0·7469	0·7486	0·7497	0·7499	0·7500	0·7500
0·20	0·5569	0·5865	0·6437	0·7020	0·7285	0·7403	0·7456	0·7491	0·7498	0·7500	0·7500
0·30	0·5448	0·5561	0·5806	0·6289	0·6934	0·7245	0·7386	0·7477	0·7495	0·7500	0·7500
0·40	0·5350	0·5446	0·5556	0·5772	0·6201	0·6865	0·7215	0·7442	0·7488	0·7500	0·7500
0·50	0·5254	0·5349	0·5444	0·5549	0·5744	0·6135	0·6803	0·7359	0·7472	0·7499	0·7500
0·60	0·5161	0·5253	0·5347	0·5440	0·5540	0·5718	0·6079	0·7159	0·7431	0·7499	0·7500
0·70	0·5070	0·5160	0·5251	0·5342	0·5433	0·5530	0·5695	0·6675	0·7334	0·7497	0·7500
0·80	0·4980	0·5068	0·5158	0·5247	0·5335	0·5424	0·5520	0·5984	0·7098	0·7493	0·7500
0·90	0·4893	0·4979	0·5067	0·5154	0·5240	0·5327	0·5415	0·5652	0·6531	0·7482	0·7500
1·00	0·4807	0·4892	0·4978	0·5062	0·5147	0·5232	0·5318	0·5500	0·5903	0·7457	0·7499
1·20	0·4642	0·4723	0·4805	0·4886	0·4967	0·5048	0·5130	0·5299	0·5475	0·7251	0·7495
1·40	0·4484	0·4561	0·4640	0·4717	0·4795	0·4872	0·4951	0·5113	0·5281	0·6183	0·7474
1·60	0·4333	0·4406	0·4481	0·4556	0·4630	0·4704	0·4780	0·4934	0·5095	0·5542	0·7346
1·80	0·4188	0·4259	0·4330	0·4401	0·4472	0·4543	0·4615	0·4763	0·4917	0·5329	0·6605
2·00	0·4049	0·4117	0·4186	0·4254	0·4321	0·4389	0·4458	0·4600	0·4747	0·5141	0·5632
3·00	0·3442	0·3496	0·3551	0·3606	0·3660	0·3715	0·3770	0·3883	0·4001	0·4316	0·4665
4·00	0·2956	0·2999	0·3043	0·3087	0·3131	0·3174	0·3218	0·3309	0·3403	0·3656	0·3936
5·00	0·2567	0·2601	0·2637	0·2672	0·2706	0·2741	0·2777	0·2849	0·2925	0·3127	0·3351
6·00	0·2255	0·2283	0·2311	0·2339	0·2367	0·2395	0·2423	0·2481	0·2542	0·2704	0·2883
7·00	0·2005	0·2027	0·2050	0·2072	0·2095	0·2117	0·2140	0·2186	0·2235	0·2365	0·2508
8·00	0·1805	0·1823	0·1841	0·1859	0·1877	0·1895	0·1913	0·1950	0·1989	0·2093	0·2208
9·00	0·1645	0·1659	0·1673	0·1688	0·1702	0·1717	0·1731	0·1761	0·1792	0·1875	0·1967
10·00	0·1516	0·1528	0·1539	0·1551	0·1562	0·1574	0·1586	0·1609	0·1634	0·1701	0·1775
12·00	0·1331	0·1339	0·1346	0·1353	0·1361	0·1368	0·1376	0·1391	0·1407	0·1450	0·1497
14·00	0·1212	0·1217	0·1222	0·1227	0·1231	0·1236	0·1241	0·1251	0·1261	0·1288	0·1319
16·00	0·1136	0·1139	0·1142	0·1145	0·1148	0·1151	0·1155	0·1161	0·1167	0·1185	0·1205
18·00	0·1087	0·1089	0·1091	0·1093	0·1095	0·1097	0·1099	0·1103	0·1107	0·1119	0·1131
20·00	0·1056	0·1057	0·1059	0·1060	0·1061	0·1062	0·1064	0·1066	0·1069	0·1076	0·1084
24·00	0·1023	0·1024	0·1024	0·1025	0·1025	0·1026	0·1026	0·1027	0·1028	0·1031	0·1035
28·00	0·1009	0·1010	0·1010	0·1010	0·1010	0·1011	0·1011	0·1011	0·1012	0·1013	0·1014
32·00	0·1004	0·1004	0·1004	0·1004	0·1004	0·1004	0·1004	0·1005	0·1005	0·1005	0·1006
36·00	0·1002	0·1002	0·1002	0·1002	0·1002	0·1002	0·1002	0·1002	0·1002	0·1002	0·1002

$$M = \frac{(FR)}{X_2 - X_e} = -0{\cdot}222\,\mathrm{h}^{-1};$$

$$t_I = \frac{X_o - X_1}{B} = 0{\cdot}0625\,\mathrm{h}, \text{ from equation (12) at } y = 0;$$

$$\beta = \frac{K}{G} = 8\,\mathrm{m}^{-1}.$$

Under the specified drying conditions one would therefore expect the moisture content of wheat to decrease from 0·75 to 0·1 kg/kg over the four following regimes:

$$\text{Regime 1}\begin{cases} X = 0{\cdot}75 - 1{\cdot}6t\,\mathrm{e}^{-8y}, \\ \text{limits}: 0{\cdot}75 > X > (0{\cdot}75 - 0{\cdot}1\,\mathrm{e}^{-8y}). \end{cases} \quad (12)$$

$$\text{Regime 2}\begin{cases} X = 0{\cdot}75 + 0{\cdot}182\,\mathrm{e}^{-8y}\{0{\cdot}449 - \exp[8{\cdot}8(t - 0{\cdot}0625)]\}, \\ \text{limits}: (0{\cdot}75 - 0{\cdot}1\,\mathrm{e}^{-8y}) > X > 0{\cdot}65. \end{cases} \quad (18)$$

$$\text{Regime 3}\begin{cases} X = 0{\cdot}55 - 1/(72\,\mathrm{e}^{-8y} + 78) + [0{\cdot}1 + 1/(72\,\mathrm{e}^{-8y} + 78)]. \\ (0{\cdot}55\,\mathrm{e}^{8y} + 0{\cdot}449)^{(0{\cdot}818\mathrm{e}^{-8y} + 0{\cdot}886)} \exp[-(7{\cdot}2\,\mathrm{e}^{-8y} + 7{\cdot}8)(t - 0{\cdot}0625)], \\ \text{limits}: 0{\cdot}65 > X > 0{\cdot}55. \end{cases} \quad (26)$$

$$\text{Regime 4}\begin{cases} X = 0{\cdot}1 + 0{\cdot}45\,\mathrm{e}^{-0{\cdot}222(t - 0{\cdot}0625)}.(0{\cdot}55\,\mathrm{e}^{8y} + 0{\cdot}449)^{0{\cdot}025}. \\ (7{\cdot}2\,\mathrm{e}^{-8y} + 8{\cdot}8)^{0{\cdot}222/(7{\cdot}2\,\mathrm{e}^{-8y} + 7{\cdot}8)}, \\ \text{limits}: 0{\cdot}55 > X > 0{\cdot}10. \end{cases} \quad (33)$$

These four equations were numerically evaluated on a CDC 3200 digital computer, the results being given in Table 1. A packed bed 2-m high was considered for this numerical example, which illustrates a number of features discussed in this paper. Drying of the bottom layer ($y = 0$) proceeded rapidly in the beginning, and in this region the first critical moisture of 0·65 kg/kg was reached in about 0·06 h, as compared with about 1·8 h at a height of 2 m. The times needed to reach the second critical moisture content of 0·55 kg/kg in the same two locations were about 0·25 h and 2 h, respectively. From Table 1 it may also be seen that the drying front started at a height above the bottom of the bed of 0·6 m and that it needed 1 h to advance to the top of the bed. The following rates of drying may be calculated from Table 1 and compared with the experimental data on which the numerical data was based [21]. The constant rate calculated for material situated at $y = 0$ was $-1{\cdot}6$ kg/kg h which checks exactly with the experimental data. The falling rate calculated for material situated at $0 < y < 2$ m and having a moisture content of about 0·51 kg/kg, was $-0{\cdot}094$ kg/kg h $\pm$ 3·5 per cent as compared with the experimental value of about $-0{\cdot}085$ kg/kg h.

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APPENDIX A

Solution of Equation (17)

By appropriate substitution in equation (12) one obtains:

$$X_I = X_o - Bt_I\,\mathrm{e}^{-\beta y}, \tag{A.1}$$

in which t_I is the time (single valued) required to reduce the moisture content from X_o to X_I (or from X_o to X_1 at height $y = 0$). The constant drying rate $(CR)(y)$ is expressed by equation (10), which with equations (17) and (A.1) leads to:

$$\left(\frac{\partial X}{\partial t}\right)_y - kX = B(kt_I - 1)\,\mathrm{e}^{-\beta y} - kX_o, \tag{A.2}$$

a general solution of which is:

$$X = X_o + \frac{BD}{k}\mathrm{e}^{-\beta y} + C_1\,\mathrm{e}^{kt}, \tag{A.3}$$

in which:

$$D = 1 - kt_I. \tag{A.4}$$

Since equation (A.2) expresses the partial differential of X with respect to t at constant y, the integration factor C_1 in equation (A.3) could be either a constant or a function of y. Substitution of X and t in equation (A.3) by their boundary values X_1 [from equation (A.1)] and t_I respectively, leads to C_1 as an exponential function of y:

$$C_1 = -\frac{B}{k}\exp\left[-(\beta y + kt_I)\right], \tag{A.5}$$

and the specific solution of equation (A.2):

$$X = X_o + \frac{B}{k}\mathrm{e}^{-\beta y}\{D - \exp\left[k(t - t_I)\right]\}. \tag{A.6}$$

APPENDIX B

Solution of Equation (19)

H_s in equation (19) may be expressed as a function of X from the total differential:

$$\mathrm{d}H_s = \frac{\partial H_s}{\partial X}\mathrm{d}X = \frac{\partial H_s}{\partial T_s}\cdot\frac{\partial T_s}{\partial X}\mathrm{d}X. \tag{B.1}$$

H_s vs. T_s is a logarithmic function [22] which may be linearized over temperature intervals of up to 30 degC [23]. Since variations in T_s are expected to be considerably smaller than 30 degC during the increasing-rate stage of drying (inspection of a psychrometric chart indicates 10 degC as a likely maximum figure), $\partial H_s/\partial T_s$ in (B.1) will be almost constant. Further, in view of the rather small variation expected for T_s, $\partial T_s/\partial X$ may also be linearized, yielding:

$$T_s = q + aX, \tag{B.2}$$

and equation (B.1) can be integrated to:

$$H_s = r + bX. \tag{B.3}$$

From equations (18, 19, B.3):

$$\left(\frac{\partial H_g}{\partial y}\right)_t + \beta H_g = \beta r + \beta b\left[X_0 + \frac{B}{k}e^{-\beta y}\left\{D - \exp\left[k(t - t_I)\right]\right\}\right], \tag{B.4}$$

a general solution of which is:

$$H_g = r + bX_o + \frac{\beta b B}{k} \times y\{D - \exp\left[k(t - t_I)\right]\}\,e^{-\beta y} + C_2 e^{-\beta y}. \tag{B.5}$$

The constant of integration C_2 may be determined from the boundary conditions, i.e. $H_g = H_i$ when $y = 0$ for all values of t, and the specific solution of equation (B.4) is:

$$H_g = r + bX_o + \left[H_i - r - bX_o + \frac{\beta b B}{k}y\{D - \exp\left[k(t - t_I)\right]\}\right]e^{-\beta y}. \tag{B.6}$$

APPENDIX C

Solution of Equation (25)

The peak rate of drying (IR) (y) may be determined from equation (A.2) at $X = X_1$, thus:

$$(IR)(y) = -BD\,e^{-\beta y} - k(X_o - X_1) \tag{C.1}$$

which, combined with equation (25), leads to the partial differential equation:

$$\left(\frac{\partial X}{\partial t}\right)_{u_b} + u_b X = (FR) + u_b X_2, \tag{C.2}$$

where the transformation:

$$u_b = \frac{BD\,e^{-\beta y} + E}{X_1 - X_2} \tag{C.3}$$

has been made, and:

$$E = k(X_o - X_1) + (FR). \tag{C.4}$$

The quantity E is constant, since (FR) is assumed constant [see Section 4.2(a)].

A general solution of equation (C.2) is:

$$X = X_2 + \frac{(FR)}{u_b} + C_3 \exp(-u_b t), \tag{C.5}$$

in which the integration factor C_3 could be either a constant or a function of u_b [cf. equation (C.2)]. The value of C_3 may be determined by introducing into equation (C.5) the conditions prevailing in the beginning of the falling-rate stage, i.e. $X = X_1$ when $t = t_1$. The y-dependent t_1 may be derived from equation (18) at $X = X_1$:

$$t_1 = t_I + \ln\left[\frac{L}{B}e^{\beta y} + D\right]^{1/k} \tag{C.6}$$

in which:

$$L = k(X_o - X_1). \tag{C.7}$$

Inserting now X_1 and t_1 [i.e. equation (C.6)] into equation (C.5) leads to:

$$C_3 = \left[X_1 - X_2 - \frac{(FR)}{u_b}\right]\left[\frac{L}{B}e^{\beta y} + D\right]^{\frac{u_b}{k}} \exp(u_b t_I), \tag{C.8}$$

and from equations (C.5) and (C.8):

$$X = X_2 + \frac{(FR)}{u_b} + v_b^{\frac{u_b}{k}}\left[X_1 - X_2 - \frac{(FR)}{u_b}\right]\exp\left[-u_b(t - t_I)\right], \tag{C.9}$$

where:

$$v_b = \left(\frac{L}{B}e^{\beta y} + D\right). \tag{C.10}$$

APPENDIX D

Solution of Equation (31)

A general solution of equation (31) is:

$$X = X_e + C_4 e^{Mt}, \tag{D.1}$$

the initial conditions being: $X = X_2$ when

$t = t_2$. Time t_2 is dependent on y, as was pointed out in Section 4.2(a) and may be derived from equation (26) at $X = X_2$:

$$t_2 = t_1 + \ln v_b^{1/k}\left[1 - \frac{u_b(X_1 - X_2)}{(FR)}\right]^{\frac{1}{u_b}}, \quad \text{(D.2)}$$

which, together with:

$$C_4 = (X_2 - X_e)\,e^{-Mt_2}, \quad \text{(D.3)}$$

produces the special solution

$$X = X_e + (X_2 - X_e)\,v_b^{-\frac{M}{k}} \times \left[1 - \frac{u_b(X_1 - X_2)}{(FR)}\right]^{-\frac{M}{u_b}} \exp\left[M(t - t_I)\right]. \quad \text{(D.4)}$$

Résumé—Le processus transitoire du séchage de matériaux granulaires dans des lits fixes est analysé et l'on obtient les équations exprimant les teneurs en humidité et les températures du matériau et du milieu désséchant en fonction du temps et de l'endroit dans le lit.

Tandis que la teneur en humidité des grains peut être exprimée à l'aide de fonctions exponentielles ordinaires de ces variables, l'humidité et la température du milieu désséchant demande en outre des solutions numériques. La théorie est établie pour le séchage des matériaux avec quatre régimes—un régime à vitesse constante, un autre à vitesse croissante et deux à vitesses décroissantes—et devrait s'appliquer à de nombreux produits dans les industries chimiques et alimentaires, avec de larges applications si on la modifie convenablement. L'application de la théorie est démontrée pour le séchage forcé d'un lit de grains de blé, en employant les résultats publiés pour évaluer les paramètres de transport appropriés aux conditions spéciales du séchage et en utilisant un calculateur numérique CDC 3200 pour calculer la teneur en humidité des grains en fonction du temps et de la position. Le programme Fortran écrit pour ce calcul peut être employé pour prédire le progrès du séchage d'autres matériaux tombant dans la catégorie de l'analyse actuelle.

Zusammenfassung—Es wird der instationäre Vorgang der Trocknung granulierten Materials im Festbett analysiert, und es werden Gleichungen für den Feuchtigkeitsgehalt und die Temperatur des Materials und des Trockenmittels als Funktion von Zeit und Ort im Bett abgeleitet. Während der Feuchtigkeitsgehalt des Granulats in Form gewöhnlicher Exponentialfunktionen der Veränderlichen dargestellt werden kann, erfordert Feuchtigkeit und Temperatur des Trockenmediums zusätzliche numerische Lösungen. Die Theorie wird entwickelt für die Trocknung von Materialien in vier Stufen—eine Stufe mit konstanter Geschwindigkeit, eine mit zunehmender und zwei mit abnehmender Geschwindigkeit—und kann für viele Produkte der chemischen und der Nahrungsmittelindustrie angewendet werden; bei entsprechender Abänderung vergrössert sich der Anwendungsbereich. Die Anwendung der Theorie wird für die Durchtrocknung eines Granularbettes aus Weizen gezeigt, wofür veröffentlichte Daten zur Bestimmung der Übergangsparameter der besonderen Trocknungsbedingungen und eine CDC 3200 Digital-Rechenmaschine verwendet wurde zur Berechnung des Feuchtigkeitsgehalts der Körner als Funktion von Zeit und Ort. Das für diese Berechnung aufgestellte Fortran-Programm kann zur Bestimmung des Trocknungsverlaufs anderer Materialien im Bereich der gegenwärtigen Analyse verwendet werden.

Аннотация—Анализируется нестационарный процесс сушки зернистых материалов в неподвижных плотных слоях. Выведены уравнения, дающие влагосодержание и температуру материала и сушильного агента как функции времени и пространственных координат. В то время как влагосодержание зерен можно выразить через обычные экспоненциальные функции этих переменных, определение влажности и температуры среды требует дополнительных численных расчетов. Теория разработана для матариалов, процесс сушки которых можно разбить на четыре стадии: первая стадия — при постоянной скорости, вторая — при увеличивающейся скорости и две остальные — при падающих скоростях сушки. Теория применима к сушке различных продуктов в химической и пищевой промышленности, и более широко, если её модифицировать надлежащим образом. Применение теории проиллюстрировано на примере непрерывной сушки зернистого слоя пшеницы, используя опубликованные данные для оценки параметров переноса, соответствующих заданным условиям сушки, а также путем применения цифровой вычислительной машины СДС 3200 для расчета влагосодержания зерна как функции времени и расположения. Программа Фортрана, составленная для данного случая, может быть использована для расчета процесса сушки других материалов с помощью предложенного анализа.